Forecasting prices in dynamic heterogeneous product markets using multivariate prediction methods

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ABSTRACT
Hedonic modeling is used to measure the product price behavior overall in high-tech markets. In a previous work, we showed the opportunity to extend the simple regression to a state space model evaluating hedonic prices from product prices. We created and tested an online estimation algorithm for those values. In that way, we can study time series of implicit prices for individual components of a range of products. In this paper, we implement and compare the hedonic model forecast performances respect to standard autoregressive models, univariate and multivariate. We find that hedonic values not only give extra information about supply market, but they can improve univariate predictions and in, certain periods, also multivariate ones. We show the correctness of algorithm using online version of it. An agent may predict prices for different products sharing a set of component, by taking into account the structure of production process. An application in a multi-agent supply chain simulation confirms the goodness of algorithm to be implemented in a future framework for online price analysis and prediction.

Categories and Subject Descriptors
H.4.2 [**Information Systems Applications**]: Type of Systems—*decision support, logistics*; F.2.1 [**Analysis of Algorithm and Problem Complexity**]: Numerical Algorithms and Problems—*computation on matrices, linear systems*; G.3 [**Mathematics of Computing**]: Probability and Statistics—*correlation and regression analysis, time series analysis*

General Terms
Algorithms, Measurement, Performance

Keywords
Agent-based modeling, dynamic pricing, forecasting structural models, hedonic price models, market modeling, oligopolistic competition, state-space model, trading agent competition

1. INTRODUCTION
Differentiated products in heterogeneous consumer markets often reveal similar price patterns, which to some extent can be explained by trends and volatility in shared product components. The observed dependencies among the prices of these products may provide an important source of information for vendors to improve the efficiency of their price forecasts, but also to infer about customers' valuations of separate product features. The latter insight can be used by manufacturers to align marketing operations, to adjust their production toward product varieties with highly valued components, and to steer procurement toward economically attractive components. An example are computers, which are offered in different product varieties based on similar, sometimes identical components, in heterogeneous consumer markets. Similar examples can be found in other high-tech markets, like mobile telephones and television sets, but also in markets for food products that share common inputs, like tomatoes or grain.

Despite these dependencies, applications of price forecast methods tend to focus on single products thus neglecting a potential source of information and foregoing potentially valuable insights into the valuation of components. The aim of this paper is to demonstrate the potential of codependencies among prices of differentiated products in forecasting using multivariate prediction methods. Then, another contribution is the comparison of the forecast results of a multivariate hedonic model with those obtained with classical vector- and univariate autoregressive models (VAR and AR). Hedonic models are specified more restrictive than VAR models, but they have as an advantage, that insights into component valuation are obtained. The more general VAR models, on the other hand, involve rapidly increasing numbers of parameters to estimate when the number of products analyzed increases. In this way, we validate the hedonic online algorithm that, in few seconds, estimates series of prices about components, with the same performances of the standard models. The latter are proverbially not time expensive, a characteristic such important in electronic commerce.

In the last decades, economic researches described the effects of product variety but not in term of the components included in the products [21]. In other fields as inventory management, the problem is analyzed from the optimization of logistic practices for parts and products [32, 26]. For instance, when the manufacturer finishes a component in his inventory, he is faced with two choices: to buy again the same component or to move the production over another

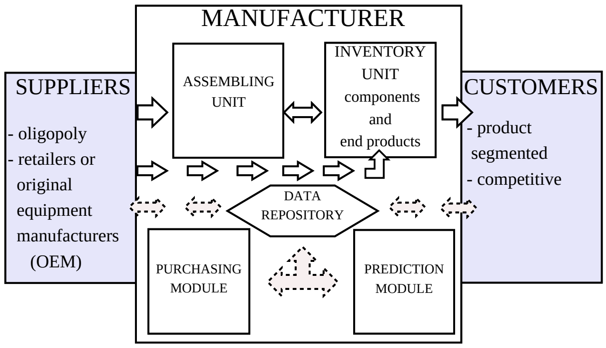

Figure 1: Forecasting agent architecture in a supply chain 2-tier for semi-durable goods. Full arrows correspond to flow of goods. Dotted gray arrows represent flows of information.

product which does not include that component. We focus on the value of the parts contained in the product prices, the hedonic value or implicit price for a component. We consider a multi-agent supply chain environment because the importance of quick forecast models based on an agent framework is much more important in these days. Furthermore, our algorithm is applicable to quick and versatile markets.

Nowadays, we forecast product prices in supply chain of semi-durable goods using exponential smoothing [16], machine learning [19], switching regimes [20], but no one research include hedonic values, while procurement market information could be useful. Until now, the advantages of a better forecast model can be restricted in finance and in commodity markets. Today the number of customers who buy via computer their products is increasing. When many suppliers, manufacturers, and customers will negotiate in electronic market, a forecast agent based on dual market information should be incorporate in a company framework. Since hedonic values are a direct consequence of assembling component design from the point of view of the customers, we believe that they are the latent variables of procurement and customer market. They move the forecast model in a multivariate direction, and it is not so usual in product prices forecasts. The algorithm to estimate multivariate hedonic prices for components (as described in [31]) is based on the Kalman filter technique [25, 13]. Hedonic value for each component can be used as input in an univariate model as extra information or included in a multivariate model based on the vector of hedonic variables. Usually standard forecast models consist in univariate and multivariate autoregressive models. Differently, we can dynamically forecast the selling prices of a range of product varieties in terms of the development of the implicit prices of shared components. After a multiple estimation of possible forecast values, we can measure their performances and to choose the best model to use in that period. For instance, we can follow the root mean square error performances on line for choosing which model should be used in the next period. Otherwise we can test a weight average of our forecast values.

In Figure 1 we see the flows of information and goods between both of our dynamic supply chain markets. In fact, manufacturer is often an intermediary not only of goods but also of information about the value of components. The latter information is usually hidden to the customer which evaluates it through the price of the end product. In some cases components may be also sold independently by suppliers on the consumer market (that is the case of several computer parts).

Data for the model application come from a multi agent simulation of a computer supply chain, the Trading Agent Competition for Supply Chain Management TAC SCM (see [7] for all the details of the simulation). Multi-agent games are an advantage for researcher who wants to test any prediction model, like in our case. Historical database of TAC SCM games contains all the information about supply chain variables (component prices, product prices, delivery prices, inventoried quantities, . . .) which changes game by game according to different agent strategies.

The paper is structured as follows. Section 2 positions our work in the similar literature. Section 3 presents the basic model specification and delineates the algorithm to estimate the implicit component prices. Section 4 discusses the data and application methodology. Section 5 concludes the paper with suggestions for further research.

2. LITERATURE REVIEW

Autoregressive integrated moving average (ARIMA), exponential smoothing and spectral domain, are only a small part of the numerous models that econometrical discipline offers [13, 4]. They are based on the assumption that previous values are informative about the future ones. If we consider multiple time series of product prices, vector autoregressive models (VAR) include correlation between products. Normally, we can use those models to forecast prices based on their performances. In this case, we have multiple choices to select and the best way is to study their previous performance and assume that it will remain the same in the future period. Agent can select and test one model observing online performances. Output forecasts depend on the model used in that period and not only on the estimation of the parameters. There are many methods to use multiple forecasts [1, 10]. Today, with the growth of innovative models as regime switching, and threshold autoregressive, prediction techniques take advantages of multiple estimates. State space model representation may help researcher to extract hedonic evaluations from time series. In [27] there are several examples of applications in supply chain management of each of those techniques applied in such contexts. Both [13] and [15] are a good introduction to state space models used to extract signals from a time series. When we want to extract information about components, factors, or latent variables, we may apply those methodologies, but they are still poorly extended to dynamic analysis of real components [15]. One of the few models including dynamic hedonic variables was the Dynamic Multiple Indicator Multiple Cause model (DYMIMIC) of Engle and Watson [11]. The latter was apply to extract information for interest rate in the housing market.

Dynamic forecast multi-agent systems technology has been tested in e-commerce and e-supply chains [20]. In such context, agents need quick instruments to improve strategies and to coordinate logistic operations. There is a trade off between complexity of the models and time computations. For example, if we assume structural changes in the parameters, regime switching models include an underlying latent variable which affects prices: the regime state variable. As our model, the estimation of underlying state variables based on historical data has to be quick and robust. With the growth of computer technology, we expect in the future an increase of forecast skills in price prediction [19].

A common approach in economics dealing with the valuation of product components by customers is the use of hedonic models, rooted in household production theory [22, 23] to evaluate consumer demand for heterogeneous products like cars, computers, apparel or washing machines. The hedonic technique is based on the assumption that quality differences between goods can be attributed to measurable characteristics, such as components and other product features. The shadow or implicit prices of these product characteristics (components) are estimated by regressing product selling prices on a relevant set of product characteristics in a sample of product varieties [31]. The hedonic technique has been applied to construct quality-corrected consumer price indices for many products [30, 14]. Overall in the high technology market hedonic regression is frequently used since there is a link between characteristic and components [2, 24, 12]. Furthermore, it is usually implemented for residential housing and real estate analysis [6, 5]; but no one really identified the importance of a dynamic estimation of hedonic values.

The advantages of the application in TAC SCM (the multi-agent simulation in supply chain context) are twofold: first, it provides a battery of games to test the hedonic technique where agents behaves following several strategies, as in the real world; second, component structure of computers is perhaps the perfect application of hedonic algorithm, since these goods are evaluated principally for their components stronger than other goods. Our considerations about results can be compared and deepen with previous works of the same kind [8]. We design and implement tools for intelligent real-time decision-making for such smart markets as described in [3].

3. METHODOLOGY

In this section we list the fundamental relations that are essential to estimate implicit prices and to compare performances of univariate and multivariate forecast models in e-commerce. Subsections 3.1 and 3.2 focus on the hedonic evaluation and the proper algorithm for multivariate case. The result is the Dynamic Multivariate Hedonic Model (DHMM) which considers all the hedonic variables at the same time. Subsections 3.3 and 3.4 show respectively the univariate and multivariate autoregressive relations. In subsection 3.5 we include individual hedonic values in a univariate autoregressive model. In this way, a multiple autoregressive model (MAR) is obtained, where hedonic information is the added value. Last part of this section is dedicated to the list of performance indexes we used to validate our model.

3.1 Multivariate hedonic model

Our knowledge of the market is collected in the vector of prices for the n end products. We call $y_{i,t}$ the price for the product i recorded on the market at time t. We suppose that each period generates a single observed product price for each product. If we have more than one price for the same product in a single period we may consider the mean value of these prices. If we have some missing values we can substitute them via an estimation method. We obtain a $n \times T$ matrix, $\mathbf{y}$, containing the information of the consumer market, where T is the last period of observation of the market. Each row of the matrix $\mathbf{y}$ is the generic vector of prices for the end product i during the whole period, that we call $\mathbf{y_i}$. Each column of the matrix $\mathbf{y}$ is the generic vector of prices for all end products at time t, and we call it $\mathbf{y_t}$. We start displaying the relationship between prices and characteristics. We refer to as the hedonic function because is a mapping from the end products to implicit prices for components. Our hedonic function is given by:

$$\mathbf{y_t} = \mathbf{D}\mathbf{z_t} + \mathbf{v_t}, \tag{1}$$

where $\mathbf{v_t}$ represents the stochastic component due to error in measuring; we assume $\mathbf{v_t}$ normally distributed with 0 mean and matrix of variance-covariance $\mathbf{\Sigma_v}$. As for the characteristic price index formulation [29] also in our hedonic model one of the most important motivation is to consider multicollinearity. According to (1) products are basically bundles of branded components, and the realized product prices can therefore be interpreted as an aggregate of implicit component prices. Moreover, we introduced a $n \times m$ design matrix $\mathbf{D}$, which maps the m component prices to the n product prices. A design matrix has $\{0, 1\}$ elements and it can be partitioned in submatrices with one element equal to 1 for each row, and with every column containing at least one non zero element. As each product is composed of a fixed set of components, this $\mathbf{D}$ is non-stochastic.

The basic idea of the multivariate hedonic model is that observed product prices vary with customer valuations of the constituting parts, and that these implicit component valuations evolve over time. Below, we introduce an $m \times 1$-vector a latent factor prices $\mathbf{z}_t$, with $m \leq n$. If the factors correspond with product components, then $\mathbf{z}_t$ contains the implicit component prices. We suppose the implicit component prices evolve in an autocorrelated, markovian, possibly non-stationary way over time:

$$\mathbf{z}_t = \Phi \mathbf{z}_{t-1} + \mathbf{w_t} \tag{2}$$

where $\mathbf{w_t} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Sigma_w})$ is an $m \times 1$-vector of random disturbances in the pricing process, which are uncorrelated over time. In Figure 2 we have represented the flows of information and their inter-relations. We see how the effects of both disturbances affect the series of product prices.

3.2 How to estimate hedonic prices?

If we consider hedonic prices as a state variable we can solve the system of (2) and (1) as a state space model and we solve it through the Kalman filter technique. Using the expectation-maximization (EM) algorithm or the Newton-Raphson algorithm we can estimate the unknown series of hedonic values together with other parameters [25, 18]. Quality of the estimation may be evaluated by means of residuals. They should be as small as the model include all the characteristics of a product. Actually, it is difficult to identify all the characteristic of a product in a design matrix and this is one of the reason that covariance matrix of measurement equation is often a larger matrix than the OLS regression covariance matrix. To choose the exact number of hedonic variables to be analyzed in the model one may follow different techniques. In some cases it is sufficient to consider only the most important characteristics of a product, while in other cases it is good to determine all of them. The problem, called *identifiability* or *observability*, is solved first of all, assuming the rank of the matrix Φ equal to m and considering a number of variables that provide small variance-covariance matrices of disturbances. It is very similar to

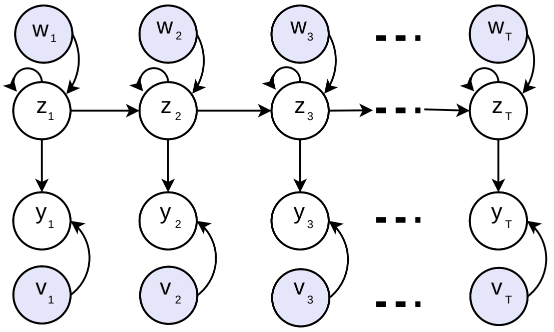

Figure 2: A directed acyclic graph with disturbances (gray circles). $\mathbf{z_t}$ is the implicit value for the vector of components. $\mathbf{y_t}$ is the series of prices for the final products. Circular arrows mean cross relations between the variables of the vector.

principal component analysis where the researcher chooses a number of components which explain sufficient variability of the data. In the supply chain context the number of physical component is well known and in the application we limit to observe only physical components of computers. Differently, if the researcher wants to include market characteristics as segmentation or discounts he should test the algorithm to avoid strange results. In the same time, we maximize the likelihood of joint distribution of prices and state variables via the expectation-maximization technique [31]. Differently from the previous work, the online version of the algorithm has a stopping rule based on the nearness of the parameters in two adjacent iterations, but a maximum number of iterations. We limit the number of iterations to allow short time of computation. The input data of algorithm are the multiple series of prices y_t in an interval $(0, T)$, the design matrix $\mathbf{D}$, and the initial values of the multi parameter $\mathbf{\Phi}, \mathbf{\Sigma_v}, \mathbf{\Sigma_w}, \mu_\mathbf{0}, \mathbf{\Sigma_0}$. Here $\mu_\mathbf{0}, \mathbf{\Sigma_0}$ represents the initial distribution parameters of hedonic state variable. After less than few seconds, output provides an estimated implicit series of the same length than product prices, and an estimate of multi parameter in the same interval. All those values represent an important instrument of market analysis in hedonic sense. Furthermore, they measure how our design matrix reflects the actual market structure. Performances of the algorithm are quasi optimal, overall under the aspect of the time of response. We will see in the application how sometimes the output parameters are not so reliable in terms of precision due on the short time of estimation. In some situations, there are other determinants for the prices different from the component evaluations. In these cases hidden Markov models could fill the lack of hedonic model computing for example switching regime parameters or time varying parameters. When the number of iterations for estimation of hedonic prices should be greater than the threshold we set, the output could be not precise. Our work wants to explore the effects of this approximations on the forecast performances.

3.3 Forecast models based simply on individual product series

The first test model is a univariate autoregressive model, AR(p), with p lag-parameter dependent on the number of periods shows high partial autocorrelation[1]. Since our data does not display long memory property we omitted a long moving average component, typical of moving averages models (MA). Autoregressive model is based on the following relation:

$$y_{i,t} = \beta_{0,i} + \beta_{1,i} y_{i,t-1} + \beta_{2,i} y_{i,t-2} + \cdots + \beta_{p,i} y_{i,t-p} + \epsilon_{i,t} \quad (3)$$

for $t = 1, \dots, T$, where T is the last value known in the series and $i = 1, \dots, n$ is the index for each type of product. We assume $\epsilon_{i,t} \sim NID(0, {\sigma_{\epsilon,i}}^2)$, or $E[\epsilon_{i,t}] = 0$ and $E[{\epsilon_{i,t}}^2] = {\sigma_{\epsilon,i}}^2$ for each $i = 1, \dots, n$. The estimation method is the OLS technique without restriction. Output to validate the model includes some statistics like t-value and significance level, the residual squared sum (RSS) and the log-likelihood. To simulate the agent online application of the model, we may compute forecast performances for many windows and for each of them we will measure performances. Forecasts of AR(p) are computed in dynamic way, using the last values known to estimate the future ones by the relation:

$$\hat{y}_{i,t+h} = \hat{\beta}^h_{1,i} y_{i,t} + \cdots + \hat{\beta}^h_{p,i} y_{i,t-p}, \quad h = 1, \dots, H. \quad (4)$$

Since an agent must predict short-medium future behavior of prices a value of $H = 40$ is convenient. In fact, our agent goal is to update the model day after day receiving information about product prices for customers, and use dynamic information for future investment in the procurement market for production planning. A value of forty days must be ideal to test also medium-long strategy for an agent.

3.4 Forecast models based on multiple series

Vector autoregressive models (VAR) take into account the co-movements among a set of variables. Each variable is regressed against its own value and all the other variables in the model for p periods back (VAR(p)). Differently from the previous model, VAR requires a lot of parameters[2] to estimate the coefficients and this is the greater disadvantage. We opted for the simplest VAR(1) to measure performance of multivariate models. Our unrestricted reduced form of the system is:

$$\mathbf{y_t} = \mathbf{\Pi y_{t-1}} + \mathbf{u_t}, \quad \text{for } t = 1, \dots, T, \quad (5)$$

where Π is the $n \times n$ matrix of coefficients constant over time and $\mathbf{u}_t \sim N(0, \mathbf{\Sigma_u})$. Note that $\mathbf{\Sigma_u}$ is constant over time and it means that OLS estimation coincides with maximum likelihood estimation (MLE). A typical output of VAR regression provides the estimates of the coefficients of Π and their standard errors. A t-value and a p-value tells us whether individual coefficients are significantly different from zero (null hypothesis). The square root of the residual variance, the sum of diagonal entries of $\mathbf{\Sigma_u}$, can be used to measure how the model fit the multivariate series. Although, to validate the model we used the coefficient of determination R^2. It represents the proportion of variation in the dependent variable that has been explained or accounted for by the regression model and can be used to measure how the model

[1] This choice is based on the analysis of partial autocorrelation functions (PACF). For example, if every product has a PACF graph shows high values (0.3-0.5) for the first three lags we can opt for $p = 3$.

[2] For instance, a VAR(1) of 16 equations requires 256 parameters as a VAR(3) for the same number of variables requires 768 of them. This is the reason why Akaike information is very important in multivariate models.

fits the multivariate series. Values of $R^2 < 0.25$, which corresponds to an $R < 0.5$, would never be acceptable.
Unfortunately, VAR models are not always adequate description of real series [17]. In our case we are assuming that every product price depend on the other ones but it may be not the case. Why an agent should be consider an increasing of price for a product if the price of another one increases or decreases? Furthermore, there is the risk that multivariate assumption is not satisfied for co-integration. This problem often invalidates the model assumption and it forces the researcher to find a more adequate formulation of the interrelationship among the variables. Obviously, all these defects affect the forecast performances and in several cases multivariate models behave worse than univariate models. Finally, we arrived to understand the importance of DHMM model as alternative to VAR model: if we use DHMM we avoid a co-integration analysis of product series, that is may be very troublesome in some cases.
Following the methodology in univariate case, we analyze VAR performances in four non-overlapping groups of estimation windows: the initial period, which collects the estimation windows 30, 35, 40, . . . 65, the initial-middle period (70, 75, . . . , 95), the middle-final period (100, 105, . . . , 135), the final period (140, . . . , 170). For all the estimation windows we compute the ahead predictions for the next $H = 40$ days.

3.5 Autoregressive forecast multiple models including hedonic values

Now, we describe a multiple model that takes into account the hedonics prices for the individual components one at a time. We start from a univariate model to which we will include hedonic information. The interpretation of this model is given by the following assumption: in certain periods a component affects the prices more than expected value. In DHMM product prices changes are only induced by hedonic vector of evaluations. We weak this assumption assuming that there exists a link with historical prices. The advantages of our new model are:

- it considers the co-dependencies between product price and the hedonic evaluation of a component included in one of the product;
- it simplifies the multivariate DHMM reducing the number of variables;
- it avoids the assumption about false interrelations amongst the product prices as in multivariate case.

As in the case of DHMM, our model is more attractive if and only if the hedonics evaluation are estimated via an optimal design matrix, and in online version it may suffer of short time of computations. We assume that our prices depend on past values but also hedonics values such that:

$$y_{i,t}^{(j)} = \beta_{0,i}^{(j)} + \beta_{1,i}^{(j)} y_{i,t-1} + \cdots + \beta_{p,i}^{(j)} y_{i,t-p} + \alpha_i^{(j)} \hat{z}_{jt} + \epsilon_{i,t}^{(j)}, \quad (6)$$

for $t = 1, \ldots, T$ and $j = 1, \ldots, m$,. Here, $(\hat{z}_{1t}, \ldots, \hat{z}_{mt})^t = \mathbf{\hat{z}_t}$ are the estimates of hedonic prices for components defined in (1). Differently from simple autoregressive model, the model in (6) includes the hedonics evaluations for components assembled into the product. For this reason, we call it multiple autoregressive model, MAR(p). Theoretically, via the equations given in (6) we state there is a link between the prices and the hedonic evaluations of characteristics of products established in (1). One time, an agent estimates the $\mathbf{\hat{z}_t}$ vector value he can plug it into one of the m forecast model to improve performances. There exist m (one for each component) MAR(p) models, that we indicate by $MAR(p)_j$, with $j = 1, \ldots, m$. In this way we can have a multiple estimation of the product value considering a different components history and hence, the complete picture of future developments. Obviously, we must choose a criteria to select the most reliable between the $m + 1$ univariate models, the VAR model, and the DHMM model to predict the actual price of one product. In Figure 3, we represented the space of forecast models limited by basic standard models where our hedonic models are positioned. In that space, many researchers have already tested other models including latent, factors, and principal component models [28, 9]. Finally, in the next section we list some indicators of forecast performances for all the models we examined in the previous subsections.

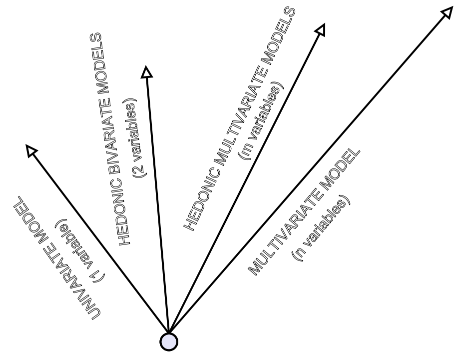

Figure 3: Our hedonic models spans from univariate and multivariate autoregressive models to test different hypothesis of co-dependencies.

3.6 Forecast performance indexes

To validate forecasts we used four indexes:

- the one day ahead relative absolute error (ODAE), the error of the model returned the next day when we found the actual price. It is good that ODAE do not exceed a fixed value selected by the agent otherwise it means that our model fails. To allow comparisons between different products we normalize them using the nominal product price. We define the index such that:

$$ODAE(i, t + 1) = \frac{|y_{i,t+1} - \hat{y}_{i,t+1}|}{np_i}, \quad (7)$$

for $i = 1, \ldots, n$ and $t = 1, \ldots, T$,. Here the values np_i are the product nominal prices obtained by a sum of nominal component costs and assembly cost as in:

$$np_i = AssCost_i + \sum_{i=1}^{numParts} NomPartCost_{i,j}, \quad (8)$$

where $NomPartCost_{i,j}$ is the nominal cost of the j-th part for good i, $numParts$ is the number of parts needed to make the good i, and $AssCost_i$ is the cost of manufacturing the good i. A nominal component cost is defined as the reference price for an individual component known from each agent at the beginning of the game. They are necessary because in this way, we can compare performances for different products in the supply chain. Reasonable values for ODAE in applications will depend on the largeness of the estimation window of the model. The longer is the series of prices the effective is the performances of the model;

- the root mean squared error computed over the h periods:

$$RMSE(i,h) = \left[\frac{1}{h} \sum_{t=1}^{h} (y_{i,t} - \hat{y}_{i,t})^2 \right]^{1/2}, \qquad (9)$$

for $i = 1, \ldots, n$, $h = 1, \ldots, H$, and $H = 40$. It gives an idea of performance in the interval of h days:

- the root mean squared positive errors, given by:

$$RMSE^+(i,h) = \left[\frac{1}{h} \sum_{t=1}^{h} \left[(y_{i,t} - \hat{y}_{i,t})^+ \right]^2 \right]^{1/2}, \qquad (10)$$

where $(x)^+ = max(0, x)$, and the root mean squared negative errors, given by:

$$RMSE^-(i,h) = \left[\frac{1}{h} \sum_{t=1}^{h} \left[(y_{i,t} - \hat{y}_{i,t})^- \right]^2 \right]^{1/2}, \qquad (11)$$

where $(x)^- = max(0, -x)$. In this way, we observe $RMSE^+$ ($RMSE^-$) to emphasize the errors inducing agent in underestimation (overestimation) of the product price. In fact, the positive (negative) error may compromise the successful of future negotiations.

To determine the accuracy of the model in more experiments we can average across days and simulations the RMSE, the $RMSE^+$, and the $RMSE^-$. Since we want an index measuring performance considering all the products, we opt for normalized prices and an average RMSE such that:

$$RMSE_A(h) = \sqrt{\frac{\sum_{i=1}^{n} \sum_{g=1}^{N_G} \sum_{s=1}^{S} \left[\frac{1}{h} \sum_{t=1}^{h} \frac{(y_{i,t}^{g,s} - \hat{y}_{i,t}^{g,s})^2}{np_i} \right]}{N_G \cdot n \cdot S}}, \qquad (12)$$

where g is the index for the simulation, N_G is the total number of simulations, s indicates the length of the time series in periods (s is the time window used for the estimation of the coefficients of the model), and S is the total number of estimation windows covered[3]. Similarly to the previous paragraph, np_i represents the nominal price of the product with index i ($i = 1, \ldots, n$). At the same way, we compute $RMSE_A^+(h)$ and $RMSE_A^-(h)$ as average over games, products, and different estimation windows. Although, since $RMSE_A(h)$ is an average we must pay attention to undervalued performance results of a model with not so low $RMSE_A(h)$. For instance, if the index for the AR(3) model is lower than the index of the $MAR(3)_1$ model, we may

[3]As we said at the end of the subsection 3.4, we split the time series in four groups of estimation windows. For each group we compute $RMSE_A(h)$ obtaining four index for the whole series.

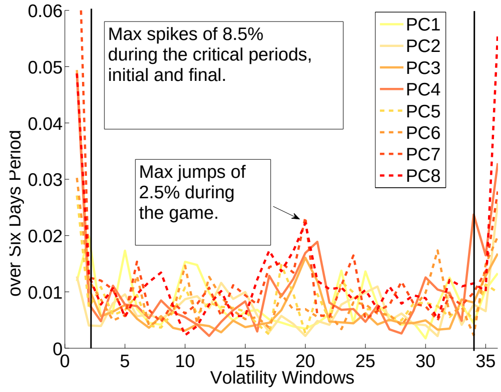

Figure 4: 6 days moving window volatility for PINTEL computer prices in 220 days. The x-axis includes $220/6 \approx 36$ windows

have in any cases that the latter performed better than the first one. We will judge a model useful and applicable if its index show similar results to the best one model for different values of h. Furthermore, we may assign one point when $RMSE(i,h)$ is larger than the same index for another model for each $i = 1, \ldots, n$ and each $h = 1, \ldots, H$. Collecting all the points we obtain a new index. It measures the relative efficacy of a model and hence the proximity of the same to another model in a period of s estimation windows. We define a score function:

$$Q_{\frac{mdl1}{mdl2}} = \sum_{s=1}^{S} \sum_{i=1}^{n} \sum_{g=1}^{NG} \sum_{h=1}^{H} I \begin{pmatrix} RMSE(i,h)_{g,mdl1} \\ RMSE(i,h)_{g,mdl2} \end{pmatrix}, \qquad (13)$$

where $mdl1$ and $mdl2$ are two forecast models and the I function assign one if $RSME(i,h)$ for one model is better then the $RSME(i,h)$ for another model for the exact day, product, game, and forecast period. We may consider points when root mean square error is simply lower than other or to give points for differences of 5% or 10%. Values of Q give us a measure of performance over a large number of cases.

4. TESTBED APPLICATION: THE TRADING AGENT COMPETITION FOR SUPPLY CHAIN MANAGEMENT

To analyze results of hedonic framework including multiple forecast models we used data from a multi agent simulation of a computer market supply chain, TAC SCM. In this virtual market agent-manufacturer produces 16 types of computers buying and assembling branded parts for motherboard, CPU, RAM and hard drives. Every component is produced in two features by each supplier (ten different suppliers provides the five components in double version). Agents may choose between PINTEL or IMD motherboards, 2Ghz or 5Ghz CPUs, 1Gb or 2Gb RAMs, 300Gb or 500Gb Hard Drives. Availability of different components and demand for computers varies randomly through the game. Data are extracted from an archive and consists of 85 games. We used 5 games for analysis and graphics, 50 games for training of both algorithms and the remaining 30 games for measure the performances. In TAC SCM equation (1) elements are $\mathbf{y}$, the vector of prices of 16 types of

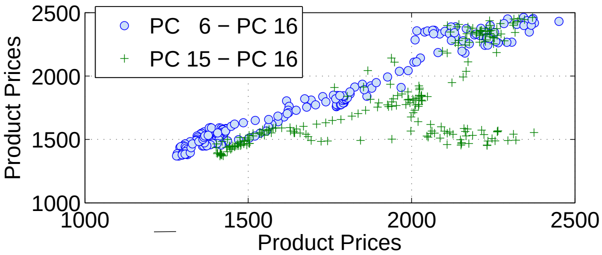

Figure 5: Scatterplots for two pairs of computers for all the days in Game 6.

computers in time $t = 1, \ldots, T$, and $T = 220$, the matrix $\mathbf{D}$ of dimension 16×5 given by:

$$\mathbf{D}' = \begin{pmatrix} 1111111111111111 \\ 0000000011111111 \\ 0000111100001111 \\ 0011001100110011 \\ 0101010101010101 \end{pmatrix}, \qquad (14)$$

and the vector of hedonic values of components $\mathbf{z}_t$. The number of characteristics that we examined is $m = 5$: the base product (included in each type of computer), the motherboard differential (between PINTEL and IMD brand), the CPU differential (between 2 GHz and 5 GHz), the Hard Drive differential (between 300 GB and 500 GB), and the RAM differential (between 1 GB and 2 GB). Each row of matrix $\mathbf{D}$ describes a single type of PC based on its physical components. If a component is included a digit 1 compares otherwise a 0 appears. We preferred to focus over physical components because we think they are the most important computer characteristics take into account from customers in their shopping. In an explorative analysis, figure 4 shows a volatility index during all the duration of a game computed in windows of 6 days. We can see how the initial and final periods are the more intensive for stochastic movements, until the values of 60-70%. For this reason, we divided the tails from the rest of the game in our analysis focus on different parts of the game. To show how is important but at the same time difficult to consider co-dependencies between the product prices we have included two examples of scatter plots in Figure 5. Products sharing four similar components, like the computers with ID 15 and 16, should be more correlated than PC6 and PC16 that share only three components over five. Instead they show a minor linear correlation. The mechanism of multivariate dependencies is usually the reason that pushes the researcher to the choice of a VAR model but we offer now an alternative instrument based on hedonic variables.

4.1 Results for standard models

In our univariate characterization, we set $p = 3$ since the partial auto-correlation function of the time series showed high values until this lag. Normally, in time series analysis this function indicates the appropriate lags in an AR(p) model [4]. We estimated the univariate model, AR(3) for several windows. In the first days of the game AR(3) model is not consistent since values are so few to estimate it correctly. Agent should prefer a simple AR(1) model in that case. After the first 10 days AR(3) starts to work and its performances showed robustness and good prediction. All

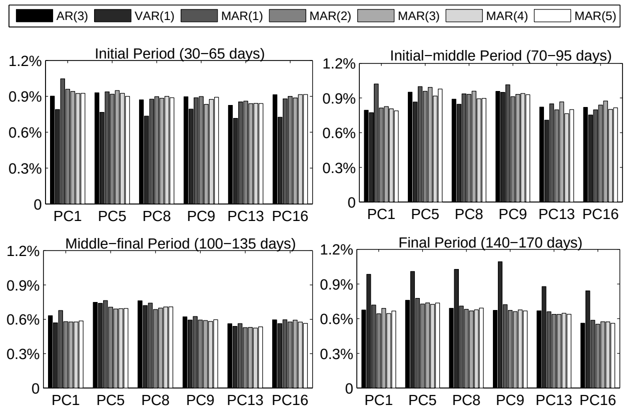

Figure 6: Average one day ahead error for different models except DHMM.

Table 1: Results of AR(3) after 40 days. Indexes in percentage for 10 games and 16 PC types. ME is the absolute mean error and SD its standard deviation

IDX	1	2	3	4	5	6	7	8
$ODAE$	0.54	0.65	1.04	1.08	0.6	0.69	0.81	1.09
ME	2.8	2.67	2.66	3.08	3.85	3.12	3.31	3.72
SD	1.66	1.47	1.83	2.13	2.41	2.17	2.21	2.17
$RMSE$	3.32	3.08	3.38	3.8	4.6	3.87	4.11	4.42
IDX	9	10	11	12	13	14	15	16
$ODAE$	0.93	0.56	1.28	0.95	0.51	0.58	0.91	1.06
ME	3.28	2.8	3.77	2.95	2.44	2.37	3.67	2.78
SD	1.57	1.55	1.84	2.06	1.55	1.5	1.72	1.32
$RMSE$	3.65	3.27	4.28	3.82	2.94	2.92	4.1	3.12

values of t-Student for estimated parameters say that coefficients are not null and for this we did not show them. All the R^2 shows an high value around 0.90-0.98. Table 1 lists the performance indexes of AR(3) in one moment of the game for an ahead period of eights days ($H = 8$). We see the full response of the model due on the correctness of the lag size even though in the initial periods of the year. In this case, we have a good result also in dynamic forecasting, with $RMSE$ at maximum at 4.6% of the nominal price. It means that in the next $H = 8$ days the root of the mean squared errors respect on nominal price of the computer is around 5%. For instance, we can forecast in the day 80 the price of PC5 for the next eight days, with a nominal cost of 2150, and the error previewed is at maximum $\frac{2.65}{21.50} \approx 57$. In the same table, we see how the performances can vary from product to product giving standard deviation of the errors. Other value of $RMSE_A$, $RMSE_A^+$, and $RMSE_A^-$ are given in Figures 7-11.

The ODAE for the univariate standard model gives optimal results also in the middle-final periods of the game. In these periods, it is lower than ODAE for the multivariate models (see plots in Figure 6). The score function (see Table 3) between AR(3) and other models shows higher performance of univariate models in last days of the game. Furthermore, AR(3) is one of the model to be included also in initial periods since it gives one time over four better results than VAR, and two times over three better results than DHMM.

In our application, VAR(1) model behaves so well to be selected the first model in our framework. The disadvantages

Table 2: VAR(1) coefficients. Diagonal entries of the Π matrix estimated after the first 40 days of 5 games (51-55)

Gm	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8
51	0.42	0.72	0.88	1	0.12	0.57	0.83	0.28
52	0.78	0.14	0.66	0.59	1.01	0.15	0.51	0.54
53	0.63	0.42	1.04	-0.08	0.56	1.28	0.3	0.43
54	0.05	0.19	0.3	0.61	0.44	0.57	0.56	0.78
55	0.56	0.1	0.08	1.08	0.9	0.25	0.53	0.41
Gm	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}	π_{16}
51	0.48	0.4	0.28	0.23	0.4	0.27	0.29	0.48
52	0.6	0.33	0.42	0.64	0.4	0.77	0.17	0.51
53	0.86	0.56	0.44	0.26	0.33	-0.08	-0.62	1.65
54	0.35	0.19	0.98	0.5	0.93	0.45	0.56	0.16
55	-0.17	0.21	1.06	0.84	0.5	1	0.3	0.54

are the complex lecture of the dynamic multipliers of matrix **Π** and the weakness at the end period of the games. Table 2 gives an idea of the variability of coefficient estimations when there are 16 independent variables. Table 2 reports the diagonal values in the matrix Π that should be lay in a ball of the unit value. In our application actual prices are not stationary and those values show often not stability. Although, the mathematical model has good performances in prediction overall when the game is almost stable. Probably, if one apply restrictions on **Π** could provide a clearer picture of the meaning of the coefficients than without restrictions. The one day ahead error for VAR is increasing after the 140 days (see Figure 6). This is due on the anomalous behavior of the agents that tend to empty their warehouses and price the products without consider its mean historical value and usual co-dependencies. The RMSE shows how multivariate models performs optimally in the initial and middle periods. But we should not use only VAR model to predict prices in that period. In fact, the score function (see Table 3) between VAR and other models shows many times (20% for HM and 25% for AR) it is better to use the latter.

4.2 Results for models including hedonic values

To estimate hedonics values we used the online version of the algorithm DHMM (see [31] for details). It provides also an estimate of the transition matrix, $\mathbf{\Phi}$, the covariance matrices, $\mathbf{\Sigma_v}$ and $\mathbf{\Sigma_w}$, and of initial distribution of hedonics prices, $\mathbf{z_0}$ and $\mathbf{\Sigma}_0$. We show forecast results for DHMM and the five models, $MAR(3)_1, \ldots, MAR(3)_5$, under the assumption that future developments of components valuation follow the estimated transition matrix $\mathbf{\Phi}$. In standard models future forecasting valuation of computer prices tends toward a stabilization due on the mean reverting effect. Thus, our agent could fall in the error to bet in a future stable distribution of product prices. Differently, the hedonic forecasting values follow a different mean reverting effect given by (2).

In Figure 6 we omitted the values for the DHMM since it provides not good performances in the first period of forecast. This is an effect of the scarce performances in some games due on the short time of computations. The same defects are not so evident if we consider one of the simplest models $MAR(3)_j$. Until the half of the game VAR model is the best model in sense of ODAE. Differently, after this period hedonic models and AR start to give optimal predictions. Figure 7 shows performance results for all models in

Table 3: Points for Q in each period of forecast. Pts 0 if $RMSE(i,h)$ is lower for the first model respect than second model. Pts α if $RMSE(i,h)$ is lower of α for the first model respect than second one

Prd	HM/AR			HM/VAR			AR/VAR
	Pts 0	Pts 5‰	Pts 10%	Pts 0	Pts 5‰	Pts 10%	Pts 0
1-25	0.43	0.14	0.09	0.19	0.01	0.00	0.26
30-65	0.35	0.14	0.08	0.16	0.01	0.00	0.29
70-95	0.36	0.13	0.08	0.16	0.01	0.00	0.30
100-135	0.39	0.13	0.07	0.20	0.01	0.00	0.32
140-170	0.35	0.11	0.06	0.40	0.13	0.09	0.53
175-200	0.33	0.07	0.03	0.47	0.18	0.23	0.67
Prd	AR/HM			VAR/HM			VAR/AR
	Pts 0	Pts 10%	Pts 50%	Pts 0	Pts 10%	Pts 50%	Pts 0
1-25	0.57	0.13	0.05	0.81	0.17	0.06	0.74
30-65	0.65	0.24	0.09	0.84	0.29	0.10	0.71
70-95	0.64	0.19	0.04	0.84	0.24	0.04	0.70
100-135	0.61	0.12	0.02	0.80	0.15	0.02	0.68
140-170	0.65	0.14	0.02	0.60	0.09	0.01	0.47
175-200	0.67	0.13	0.01	0.53	0.06	0.00	0.33

four groups of forecast windows compared to the other models, using the index $RMSE_A(h)$ as in (12). We tested the models across time series of length $30, 35, \ldots, 60, 65$ in the first graph (initial period), $70, 75, \ldots, 95$ in the second graph (initial-middle period), $100, 105, \ldots, 135$ in the third graph (middle-final period), $140, 145, \ldots, 170$ in the fourth graph (final period). We see how VAR(1) performances are almost always the best ones. Our hedonic model has a strangely behavior during the first ahead days of forecast since its estimates some times are not sharp. Although, the same values are quite similar for AR(3) and $MAR(3)_m$. It confirms the hypothesis of good performances of bivariate models not so different respect than multivariate ones. Hedonics information may improve the forecasts in several situations as in the middle-final days of the game. Both bivariate and multivariate hedonic models improve forecasts around the 1-2% respect than standard autoregressive models, corresponding in an average absolute value difference of 20-50 per unit produced. Hence, a simple inclusion of hedonic information may improve the forecast price agent framework. We see how all models improve with the increasing time series length. After 150 days performances are quite similar except first days forecast of multivariate models. Our agent should prefer to use a different model depending on the period of the game and on the ahead days of forecasts. Thus, our hedonic multivariate model performs very well in certain games and periods and it is better then VAR(1) in the last days of the game. Failures of the model are due on the short time of computations in online conditions, the larger number of parameters to be estimated than other models and on the non-observability of hedonic variables. The larger the number of variables is in the model the larger probability to have forecast errors.

In Table 3, points obtained from DHMM are compared to AR and VAR scores. The good performances of hedonic model, when algorithm achieves to estimate perfectly the component implicit price behaviour, pass from 16% to 47% in the last periods. What can we say about symmetrical property of our forecasts? In Figure 8 we see some differences in performance that convince us to build a framework considering all the models. For instance, AR in the first pe-

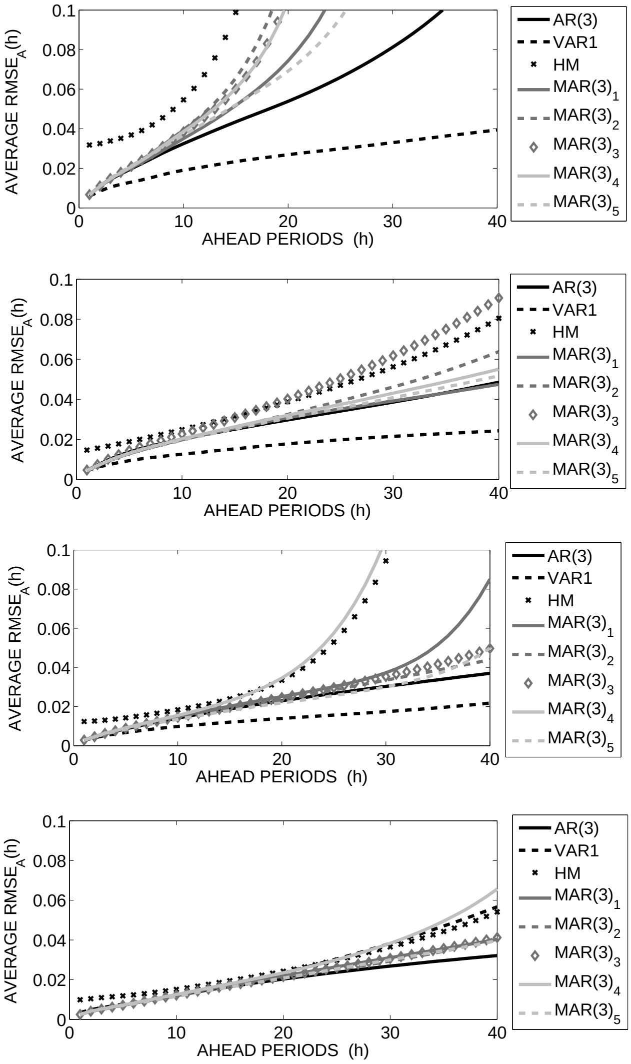

Figure 7: $RMSE_A(h)$ in four group of windows. From up to bottom: (1) initial period 30-65 days; (2) initial-middle period 70-95 days;(3) middle-final period 100-135 days; (4) final period 140-170 days

riods tends to overestimate prices. Differently, multivariate models in the last periods tends to underestimate prices. How agent can weight the multiple forecast information to point in the right direction to prediction of prices? Our framework includes multiple indicators of market trend that could substitute the standard autoregressive estimators. The latter in non stationary markets suffer from the defect of the reversion to the mean.

5. CONCLUSIONS AND FUTURE WORK

We designed and implemented our multivariate hedonic algorithm for online forecasting in a dynamic heterogenous market. Furthermore, we developed a method for comparative performance assessment of various univariate and multivariate forecast methods. Our multivariate methodology describes and compares a set of instruments that an agent may implement in electronic markets for dynamic pricing. Multivariate analysis is surely an advantage in this context.

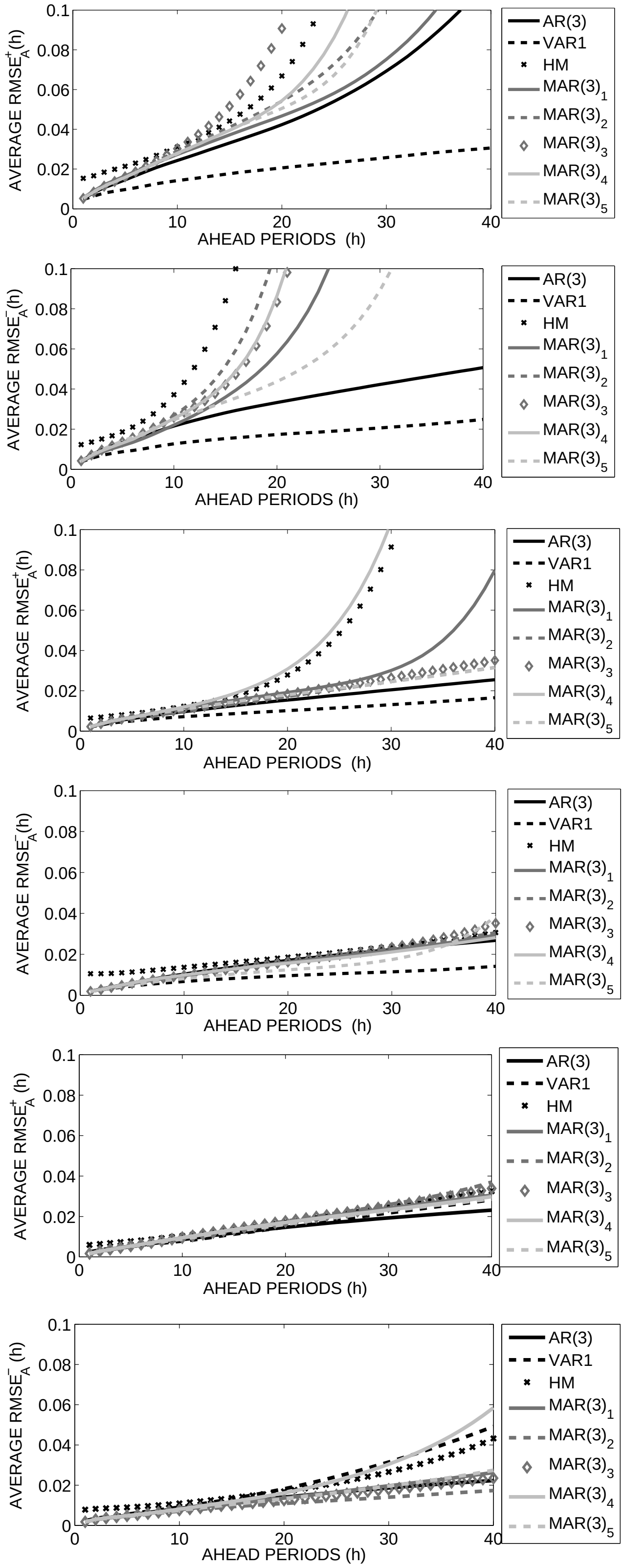

Figure 8: $RMSE_A^+$ and $RMSE_A^-$ in three groups of windows. From up to bottom: (1)-(2) initial period 30-65 days; (3)-(4) middle-final period 100-135 days; (5)-(6) final period 140-170 days.

Then, we found that hedonic model may fill the gap of standard multivariate errors over all in specific intervals. In fact, including hedonic models, an agent can examine a multiple choice of predictions about future prices based on components and historical product prices. Robustness of AR and VAR models together with the correct performances of hedonic models can improve a forecast multiple framework. This is one way to use hedonic information. In our future research, we want to test connections between procurement prices and hedonic prices. In such circumstance, hedonic values can be used also as a predictor of unobservable component prices. So, we can consider them as predictors of both supply chain market prices.

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